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AC becomes nearer equal to its projection on AB . Just so BC , when C is at A (or C) $AC+BC$ =the sum of their projections on AB , or $AO+OB$.

That is, as C moves from the middle of the arc ACB , $AC+BC$ becomes less. $\therefore AB+BC$ is a maximum when C is at the middle of the arc ACB .

IV. Solution by B. F. YANNEY, A. M., Professor of Mathematics, Mount Union College, Alliance, Ohio.

The required point is the mid-point of the semi-circumference.

Proof. Let Q be any other point in the semi-circumference than point P .

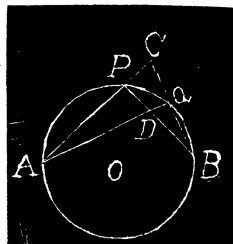
Produce AP and BQ until they meet, as at C .

Now $\triangle APD = \triangle BPC$, each being a right triangle, and having a leg and an acute angle of the one = to a leg and a homologous acute angle of the other.

$\therefore PC = PD$.

But $AC > AQ$, and $DB > QB$. Hence $AP + PD + DB > AQ + QB$, or $AP + PB > AQ + QB$.

Q. E. D.



Other solutions were furnished by NELSON L. RORAY, CHARLES C. CROSS, M. A. GRABER, J. M. COLAW, and COOPER D. SCHMITT. Professors Cross, Colaw, and Yanney each furnished two solutions.

CALCULUS.

78. Proposed by COOPER D. SCHMITT, A. M., Professor of Mathematics, University of Tennessee, Knoxville, Tenn.

Investigate value of $\left(\frac{\tan x}{x}\right)^{1/x^n}$ where x is 0 and n has consecutive values 1, 2, 3, 4, ... Is there any law governing the different results? When $n=1$, result is 1; when $n=2$, result is e^3 ; $n=3$, gives ∞ , etc.

II. Solution by the PROPOSER.

Let $y = \left(\frac{\tan x}{x}\right)^{1/x}$, making $n=1$, in the given expression.

Then $\log y = \frac{\log(\tan x/x)}{x}$ which is of the form $0/0$.

$$\text{By the calculus this} = \frac{\frac{x \sec^2 x - \tan x}{x^2}}{\frac{\tan x}{x}} = \frac{x \sec^2 x - \tan x}{x^2} \Bigg]_{x=0} \text{ since } \frac{\tan x}{x} \Bigg]_{x=0} = 1.$$

The numerator of this expression is the fundamental form which will occur in all the different values given to n .

Continuing the evaluating process we have

$$\left. \frac{x \sec^2 x - \tan x}{x^2} \right]_{x=0} = \left. \frac{2x \sec^2 x \tan x}{2x} \right]_{x=0} = \left. \sec^2 x \tan x \right]_{x=0} = 0.$$

Hence $y = e^0 = 1$.

Let $n=2$, and we have as before,

$$\left. \frac{x \sec^2 x - \tan x}{2x^3} \right]_{x=0} = \left. \frac{\sec^2 x \tan x}{3x} \right]_{x=0} = \left. \frac{\sec^4 x + 2 \sec^2 x \tan^2 x}{3} \right]_{x=0} = \frac{1}{3}.$$

Hence $y = e^{\frac{1}{3}}$.

Let $n=3$, and we have

$$\left. \frac{x \sec^2 x - \tan x}{3x^4} \right]_{x=0} = \left. \frac{\sec^2 x \tan x}{6x^2} \right]_{x=0} = \left. \frac{\sec^4 x + 2 \sec^2 x \tan^2 x}{12x} \right]_{x=0} = \frac{1}{0} = \infty.$$

Then $y = e^\infty = \infty$.

Let $n=4, 5, 6, \dots$ and we have

$$\left. \frac{x \sec^2 x - \tan x}{4x^5}, \quad \frac{x \sec^2 x - \tan x}{5x^6}, \quad \dots, \quad \frac{x \sec^2 x - \tan x}{nx^{n+1}} \right]_{x=0}$$

The numerator will remain as in the former cases, but the denominator will have increasing powers of x . Hence the result will always be ∞ .

III. Note by Dr. E. D. ROE, Jr., Norwood, Mass.

In connection with this problem attention might perhaps be called to the fact the indeterminate form 1^∞ may often be more advantageously dealt with by referring it directly to the foundation of the form, viz, to

$$\lim_{\varepsilon \rightarrow 0} (1 + \varepsilon)^{\frac{1}{\varepsilon}} = e,$$

where ε is any expression whatever that $\rightarrow 0$, than by the usual method given in the books. Thus in the given example,

$$\frac{\tan x}{x} = 1 + \frac{x^2}{3} (1 + \varepsilon),$$

where ε is a convergent series and therefore $\rightarrow 0$, as $x \rightarrow 0$.

$$\left(\frac{\tan x}{x} \right)^{\frac{1}{x}} = \left\{ \left[1 + \frac{x^2}{3} (1 + \varepsilon) \right]^{\frac{3}{x^2 (1 + \varepsilon)}} \right\}^{\frac{1 + \varepsilon}{3x^{n-2}}},$$

within the limits of convergence and certainly for small values of x . Therefore

$$\begin{aligned} \lim_{x \rightarrow 0} \left(\frac{\tan x}{x} \right)^{1/x^n} &= \lim_{x \rightarrow 0} \left\{ \left[1 + \frac{x^2}{3} (1 + \epsilon) \right]^{x^2 (1 + \epsilon)^{-1}} \right\}^{\frac{1 + \epsilon}{3 x^{n-2}}} = e^{\lim_{x \rightarrow 0} \frac{1 + \epsilon}{3 x^{n-2}}} \\ &= \left(\epsilon \frac{1}{3 x^{n-2}} \right)_{x=0}. \end{aligned}$$

This has the advantage of being a direct, closed expression, which shows at once the particular results, $1, e^{\frac{1}{3}}, \infty$, when n equals $1, 2, 2+r$, where r is any positive integer, and also the general law for any value of n , of whatever character it may be.

MECHANICS.

73. Proposed by J. K. ELLWOOD, A. M., Principal of Colfax School, Pittsburg, Pa.

A sixteen foot plank weighs thirty-two pounds and is supported by two props, four feet and two feet from the ends. What weight is supported by each prop?

Solution by C. HORNUNG, A. M., Professor of Mathematics, Heidelberg University, Tiffin, Ohio; P. S. BERG, B. Sc., Principal of Schools, Larimore, N. D.; J. SCHEFFER, A. M., Hagerstown, Md.; CHARLES C. CROSS, Libertytown, Md.; ELMER SCHUYLER, High Bridge, N. J.; A. H. BELL, Hillsboro, Ill.; CHARLES E. MEYERS, Canton, O., and M. A. GRUBER, A. M., War Department, Washington, D. C.

Let x = the number of pounds supported by the prop four feet from the end. Then taking moments around the other point of support, we have:

$$(6+4)x = 6 \times 32, \text{ or } x = 19.2 \text{ pounds.}$$

The other prop supports $32 - 19.2 = 12.8$ pounds.

Also solved by G. B. M. ZERR.

74. Proposed by B. F. FINKEL, A. M., M.Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

In the experiment of swinging in a vertical circle a glass containing water, and suspended by means of a string, if the string be two feet long, what must be the velocity at the lowest point if the experiment is to succeed? [From *Ziwet's Theoretical Mechanics*, Part III., p. 96.]

Solution by the PROPOSER.

Let M = the mass of the glass and water, f = the centrifugal force, r = 2 feet = radius. Then $f = Mv^2/r$.

At the highest point, in order that the experiment may be successful $f = Mg$.
 $\therefore Mv^2/r = Mg$, whence $v = \sqrt{rg}$.

The velocity, v_1 , due to gravity in passing from the highest to the lowest point is $v_1 = \sqrt{(2gs)}$, where $s = 2r$.

Hence, the velocity, v_2 , at the lowest point is,

$v_2 = \sqrt{(v^2 + v_1^2)} = \sqrt{(rg + 2gs)} = \sqrt{(321.6)} = 17.94$ feet per second,
 g being equal to 32.16.

Also solved with slightly different results by G. B. M. ZERR, and CHARLES E. MEYERS.